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Dimensional Analysis of Jet-Noise Data

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A formula for scaling laboratory model jet-noise data to that of a full-size jet engine is established. This is done by dimensional analysis and the Buckingham Π theorem. It is well known that the noise intensity radiated by a jet varies as a high power of its velocity. This power-law dependence is supported by theory and experiment. The most famous power law is the Lighthill U^8 law. In this work, a dimensionless form of the power law is sought and developed. Through the use of this nondimensional power law, a "hot-jet" limit is found at high jet temperature. In the hot-jet limit, the velocity exponent and the proportionality constant of the power law are insensitive to further increase in jet temperature. This is a somewhat surprising result. Experimentally, it has been observed that seasonal temperature variation can lead to a 1.5- to 2.0-dB change in jet noise intensity. Here, environmental effects on jet noise such as seasonal and altitude variations are studied through results established by dimensional analysis. Finally, a way to scale jet mixing noise from perfectly expanded jets to shock-containing jets is proposed.

Nomenclature

a_j = speed of sound of fully expanded jet
a_∞ = ambient sound speed
c_p = specific heat at constant pressure
D = nozzle exit diameter
D_j = fully expanded jet diameter
f = frequency
I = noise intensity
L = fundamental dimension of length
M = fundamental dimension of mass
M_d = nozzle design Mach number
M_j = fully expanded jet Mach number
p_e = pressure at nozzle exit
p_j = fully expanded pressure of jet
p_∞ = ambient pressure
R = gas constant
Re = Reynolds number
r, θ, ϕ = spherical polar coordinates
S = power spectral density
T = fundamental dimension of time
T_j = fully expanded jet temperature
T_r = reservoir temperature of jet
T_∞ = ambient temperature
γ = ratio of specific heats
ν = kinematic viscosity
ρ_j = fully expanded gas density of jet
ρ_∞ = ambient gas density

Subscript

e = variable at the nozzle exit

I. Introduction

THE primary objective of this work is to develop scaling formulas to allow model laboratory jet-noise data to be used for full-scale engine design and prediction. A secondary objective is to investigate, through dimensional reasoning and experimental correlations, the influence of environmental variables such as ambient temperature and pressure on jet-noise intensity.

In the past, there have been a number of investigations devoted to the establishment of scaling methods and correlation formulas for jet noise.[1–4] These efforts generally follow two approaches. The first approach uses primarily empirical correlations based on intuition and physical insight for the development of scaling parameters. The work of Stone et al.[5,6] appears to be most representative of this approach. The second approach makes use of analytical results of idealized models to derive variable groups by which one attempts to collapse large sets of data. The acoustic analogy is, invariably, the starting point of most of the analytical models. Classical formulas such as Doppler frequency and convective amplification factor, with appropriate modifications, are favorite correlation variables employed. The work of Ahuja[7] and Gaeta and Ahuja[8] is an exemplary contribution of this line of approach.

In the present investigation, dimensional analysis and the Buckingham Π theorem are used to establish a basic scaling formula. An extensive search in the literature indicates that such an approach has not been attempted before. The most important advantage of dimensional analysis is simplicity. Detailed noise-source distribution and sound-generation processes do not need to be known. It is also not necessary to solve any equations, especially systems of partial differential equations such as the Navier–Stokes equations. On the other hand, results obtained through dimensional analysis are of limited value. It sheds no light on the physical processes involved. Generally speaking, it offers no more than gross scaling formulas. It works well only when the number of input variables is small; otherwise no simple useful relationship can be derived. Dimensional analysis, however, can become a more powerful tool when supplemented by experimental observations or by partial analytical results. This is what we intend to do throughout this study.

It is also worthwhile to mention that in engineering applications, a one-third octave band is a widely used frequency unit. As a result, one-third octave band noise spectra have been used to correlate jet-noise data from different-sized models. For example, Viswanathan[9] has been successful in collapsing jet-noise spectra from nozzles of several sizes over the entire measured frequency range. There is no question that this represents an excellent engineering endeavor. However, it must be pointed out that one-third octave band is a dimensionally ill-defined variable. As such, it is not an acceptable scientific variable and definitely plays no role in dimensional analysis. For noise prediction and data analysis purposes, such variables should be used only with extreme care. In this work only narrow-band noise data are used.

The rest of this paper is organized as follows. In Sec. II, dimensional analysis is applied to an ideal jet-noise experiment. Input variables are established and dimensionless groups are formed. A scaling formula is then developed through the Buckingham Π theorem. This scaling formula is put to the test in Sec. III. Many sets of experimental measurements are used to demonstrate its scaling

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capability. Section IV is devoted to the development of a dimensionless power law for jet-noise intensity. This effort is supplemented by extensive empirical data analysis. It is observed that at a sufficiently large jet-temperature ratio, the velocity exponent and the proportionality constant of the power law become insensitive to further increases in jet-temperature ratio. The existence of this "hot jet" limit is quite surprising. In Sec. V, the effects of environmental variables such as ambient temperature and pressure on the radiated noise intensity are investigated. It is shown that there is a difference between noise measurements in the summer and in the winter. A seasonal variation in jet-noise intensity as large as 1.5–2.0 dB is to be expected. Finally, in Sec. VI, the possibility of scaling turbulent mixing noise of shock-containing supersonic jets to that of perfectly expanded jets is investigated. A scaling formula is proposed and tested. It will be shown that there is a good collapse of data over a wide range of Mach numbers and jet temperature ratios. A short discussion of the issue of jet-noise data quality serves as the conclusion of this paper.

II. Dimensional Analysis of an Ideal Jet-Noise Experiment

We will consider an ideal jet-noise experiment, as shown in Fig. 1. We will define an ideal experiment as one that satisfies the following conditions/assumptions:

1) The jet velocity profile at the nozzle exit is fairly uniform and the nozzle internal-wall boundary layer is reasonably thin, so that it has no strong effect on the radiated jet noise.

2) The level of upstream disturbances is normal and exerts no unusual influence on the noise of the jet.

3) Noise absorption by ambient air (humidity absorption) can be ignored or the measured data have been adjusted. In other words, only the lossless far-field noise is considered.

Conditions/assumptions 1 and 2 have the explicit purpose of restricting our study to pure jet mixing noise. Here, to avoid misunderstanding, we wish to make it clear that an ideal experiment is not an impossible experiment. Recently, Viswanathan and Clark[10] studied the effects of internal nozzle boundary layer thickness on jet noise. They concluded that unless there was flow separation inside the nozzle, boundary-layer thickness had minimal impact on pure jet mixing noise. Condition/assumption 2 is extremely important. It has been known since the 1970s that upstream disturbances such as tones could have drastic effect on the level and spectral shape of the noise of a jet.[11–17] In a more recent study, Viswanathan[18] documented the contamination of jet noise by various components of rig noise in a not properly designed test facility. But once rig noise is sufficiently reduced, the quality of far-field jet noise is significantly improved, opening the possibility of scalability. Assumption 3 is needed because humidity correction is frequency and hence jet size specific. Such corrections cannot be scaled.

We will refer to the variables of an experiment that affect the noise radiated from a jet as input variables. We will call variables that are a part of the measured or processed data of the experiment as output variables. The far-field noise radiated from a jet is influenced by both the jet exit variables and the variables characterizing the ambient conditions. The ambient variables as shown in Fig. 1 are p_∞, ρ_∞, and T_∞. But instead of T_∞, we may use a_∞, the ambient sound speed. The two variables are equivalent since $a_\infty = (\gamma R T_\infty)^{1/2}$. Also, by the equation of state of a perfect gas, we have $\rho_\infty = \gamma p_\infty / a_\infty^2$, so that only two of the three ambient variables are independent. We will take p_∞ and either a_∞ or T_∞ as the input ambient variables.

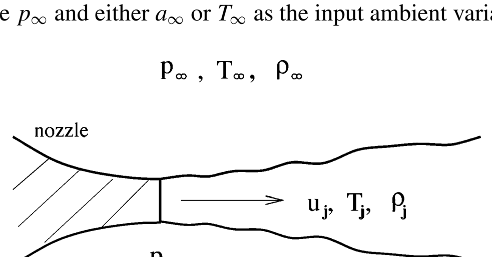


Fig. 1 Input variables of an ideal jet-noise experiment.

The jet variables are U_j (fully expanded jet velocity), p_e (pressure of jet at the nozzle exit), ρ_j, T_j (fully expanded jet density and temperature), ν (kinematic viscosity), and D_j (fully expanded jet diameter; in the case of subsonic and perfectly expanded supersonic jets, it is the same as D, the nozzle exit diameter). For subsonic and perfectly expanded supersonic jets, the static pressure of the jet at the nozzle exit is almost the same as the ambient pressure; that is, $p_e \cong p_\infty$. Thus p_e is not an independent input variable. We will defer the consideration of imperfectly expanded supersonic jets for which p_e is not equal to p_∞.

Again, by the perfect gas law, we have $\rho_j = p_\infty/(RT_j)$. It follows that ρ_j is also not an independent input variable. In a jet-noise experiment, it is more convenient to control the reservoir temperature, T_r, of the jet. By the energy equation, T_j is related to U_j and T_r by

$$C_p T_r = C_p T_j + \tfrac{1}{2} U_j^2$$

Hence, we may use T_r instead of T_j as an input variable. Therefore, the input variables characterizing the jet exit conditions are U_j, T_r, ν, and D_j.

Let the three fundamental dimensions in mechanics, namely, mass, length, and time, be denoted by M, L, and T, respectively. The dimensions of all the input variables are

$$p_\infty(M/LT^2), \qquad a_\infty(L/T), \qquad U_j(L/T)$$

$$T_r(L^2/T^2R), \qquad \nu(L^2/T), \qquad D_j(L)$$

where R is the gas constant of the equation of state. It is to be noted that of the six variables, only p_∞ has dimension M. For this reason, it cannot be combined with other input variables to form a dimensionless group. Three independent dimensionless groups can be formed from the remaining five variables. A convenient choice of groupings leads to

$$U_j/a_\infty, \qquad T_r/T_\infty, \qquad U_jD_j/\nu = Re \text{ (Reynolds number of the jet)}$$

Of interest in a jet-noise experiment is the far-field noise-power spectral density S at a point with spherical coordinates (r, θ, ϕ) (the origin of the coordinate system is at the center of the nozzle exit), defined by

$$\overline{p^2} = \int_0^\infty S(r, \theta, \phi, f)\,\mathrm{d}f \tag{1}$$

where $\overline{p^2}$ is the time average of the square of the pressure fluctuations at (r, θ, ϕ) and f is the frequency. A simple dimensional analysis of Eq. (1) indicates that the dimensions of S are $(M^2L^{-2}T^{-3})$. At the same time, f has the dimension of T^{-1}. On using the input variables to nondimensionalize the two output variables, two dimensionless groups can be formed. They are

$$S\big/p_\infty^2(D_j/U_j), \qquad fD_j/U_j \text{ (Strouhal number)}$$

Now according to the Buckingham Π theorem,[19–21] the dimensionless power spectral density must be a function of all the other dimensionless groups. Therefore, we may write

$$\frac{SU_j}{p_\infty^2 D_j} = \frac{F(U_j/a_\infty, T_r/T_\infty, fD_j/U_j, Re, \theta)}{(r/D_j)^2} \tag{2}$$

In Eq. (2), the inverse-square dependence of sound on the distance of propagation is explicitly exhibited. For full-scale jet engines, the jet Reynolds number is in the millions. At such a large value, the function F would be insensitive to the exact value of the Reynolds number. Another way of stating this is that F would have essentially reached its asymptotic value for large Re, so that it is practically independent of Re. This reduces (2) to

$$\frac{SU_j}{p_\infty^2 D_j} = \frac{F(U_j/a_\infty, T_r/T_\infty, fD_j/U_j, \theta)}{(r/D_j)^2} \tag{3}$$

In a laboratory experiment, if the Reynolds number is sufficiently large, say half a million or more, Eq. (3) would also apply. When this is true, Eq. (3) provides a basic formula for scaling small-scale jet-noise data to a full-sized prototype.

If, in a series of experiments, the ambient pressure is practically the same, then we may replace p_∞^2 on the left-hand side of Eq. (3) by p_{ref}^2, where p_{ref} is the reference pressure for the decibel scale:

$$\frac{SU_j}{p_{\mathrm{ref}}^2 D_j} = \frac{\bar{F}(U_j/a_\infty, T_r/T_\infty, fD_j/U_j, \theta)}{(r/D_j)^2} \tag{4}$$

However, we must caution that Eq. (4) is just an approximation. Equation (3) is the correct scaling formula, based on dimensional analysis.

III. Scaling of Experimental Data

Scaling formulas (3) and (4) will now be applied to a variety of jet-noise data. The purpose is to see if, indeed, data can be collapsed according to these formulas. Also, if there are problems in collapsing the data, we wish to examine what might be the cause of the problems.

Figures 2a and 2b show two sets of data from Mach 2.0 jets measured by Seiner (see Ref. 22). One jet has an exit diameter of 3.60 in. (9.14 cm), the other of 1.96 in. (4.98 cm). The spectrum data are plotted according to formula (4) as a function of the Strouhal number. As can be seen from these figures, there is a good collapse of data in all directions of radiation in spite of the fact that the smaller jet is only slightly over 50% of the size of the larger jet. At Strouhal numbers above 6.0, the spectra of the smaller jet drop off abruptly. This is, most likely, an instrumentation problem. This part of the spectra should be disregarded.

Figures 3a and 3b show similar comparisons for two sets of Mach 0.5 data at a jet-temperature ratio of 2.15, measured by T. R. S. Bhat of the Boeing Company (personal communication, 2001). In this case, the larger jet is 2.3 times larger. Over the angular sector from $\theta = 50$ deg to $\theta = 160$ deg, the spectra collapse fairly well, except at very low and very high Strouhal numbers. The problem at low Strouhal number, where the spectra of the large jet make a steep dropoff, is due primarily to the low-frequency filter. This part of the spectra should be ignored. At high Strouhal number, the noise spectrum of the smaller jet is consistently lower. Figures 4a and 4b show similar data at Mach 1.0. Again overall, the spectra seem to scale quite close to each other. But the collapse again is not good at very low and very high Strouhal numbers. The low-Strouhal-number problem is the same as the Mach 0.5 jet. The discrepancy at high Strouhal numbers is somewhat puzzling. A very appealing first suggestion is that it is a Reynolds-number effect. The Reynolds numbers for the Mach 0.5 jets are 1.87×10^5 and 4.30×10^5, respectively. The smaller jet has such a low Reynolds number that it is most likely that the initial boundary layer is transitional. On the other hand, the larger jet, having a Reynolds number close to half a million, should be nearly fully turbulent. One would therefore expect that the larger jet would generate more high-frequency noise. However, for the Mach 1.0 jets, the Reynolds numbers are 4.36×10^5 and 1.0×10^6, respectively. These Reynolds numbers are sufficiently large so that Reynolds number should not be a factor in the radiated noise. But Figs. 4a and 4b (as well as Fig. 2) show significant difference in the noise spectra of the jets at high Strouhal number, very similarly to those of Figs. 3a and 3b. This casts doubt on the proposition that Reynolds number is really the cause of the discrepancy. We are, at the present time, unable to find a good explanation.

Recently, Viswanathan[9] measured a set of high-quality jet-noise data over a wide range of Mach numbers and temperature ratios. In his experiment, nozzles of 1.5-, 2.45-, and 3.46-in. (3.81-, 6.22-, and 8.79-cm) diam were used. Here we present a small set of his data using Eq. (4) as the scaling formula. Figures 5 and 6 are noise spectra at $T_r/T_\infty = 1.8$. Mach 0.6 data are shown in Fig. 5, whereas Mach 1.0 data are given in Fig. 6. Figures 7 and 8 show similar spectra at a much higher temperature ratio of 3.2. It is evident from these figures that the noise spectra from the two larger diameter nozzles do collapse well. There are some differences in the high-frequency parts of the spectra at low-Mach-number and low-temperature-ratio jet operating conditions. The spectra of the 1.5-in. (3.81-cm)-diam jet are consistently lower at high Strouhal number. The fact that the spectra of the two larger nozzles collapse nearly into a single curve suggests that the collapsed spectrum should be very close to the pure jet-noise spectrum under the particular jet operating conditions.

The reason that the spectra of the smallest jet of Viswanathan's data are consistently low at high Strouhal number is not known. It is not completely a Reynolds-number problem. Tables 1 and 2 show

Table 1 Jet Reynolds number at $T_r/T_\infty = 3.2$

$M_j \backslash D_j$	1.5 in. (3.81 cm)	2.45 in. (6.22 cm)	3.46 in. (8.79 cm)
0.6	1.4×10^5	2.38×10^5	3.36×10^5
1.0	2.67×10^5	4.50×10^5	6.37×10^5

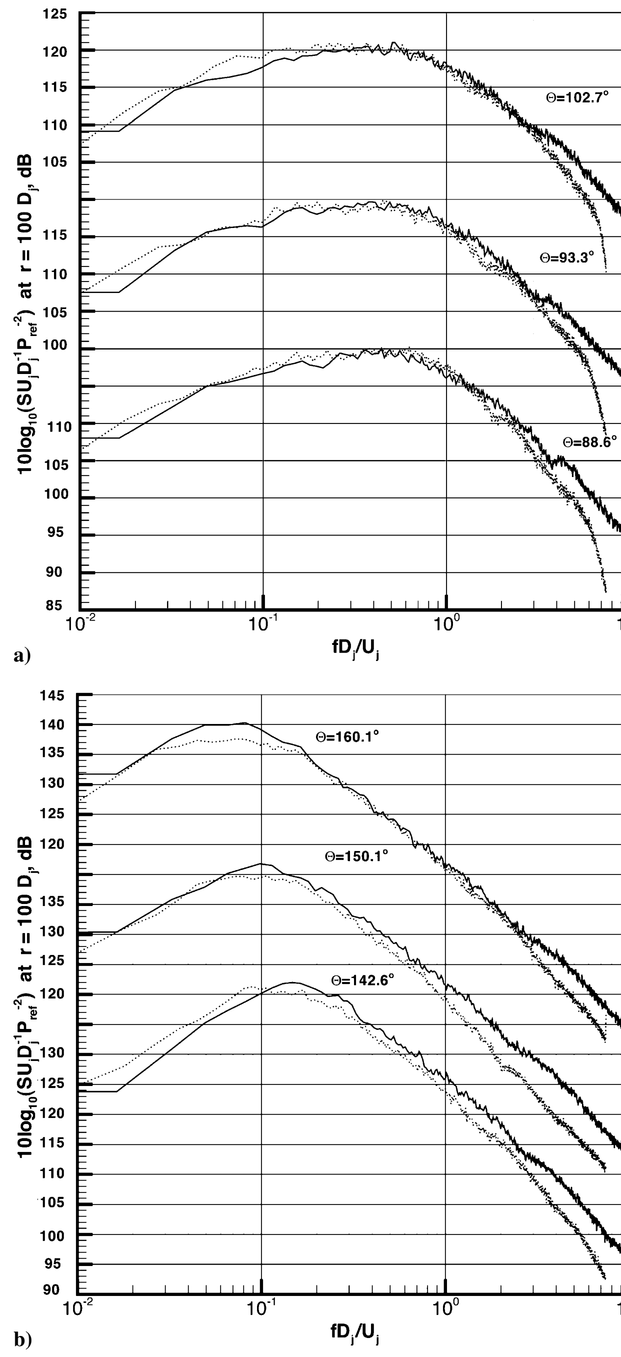


Fig. 2 Scaling of jet-noise spectra. Data from Tam et al.[22]: $M_j = 2.0$; $T_r/T_\infty = 1.8$; ——, $D_j = 3.60$ in. (9.14 cm); and ····, $D_j = 1.96$ in. (4.98 cm).

Table 2 Jet Reynolds number at $T_r/T_\infty = 1.8$

$M_j \backslash D_j$	1.5 in. (3.81 cm)	2.45 in. (6.22 cm)	3.46 in. (8.79 cm)
0.6	2.70×10^5	4.59×10^5	6.49×10^5
1.0	5.20×10^5	8.80×10^5	1.24×10^6

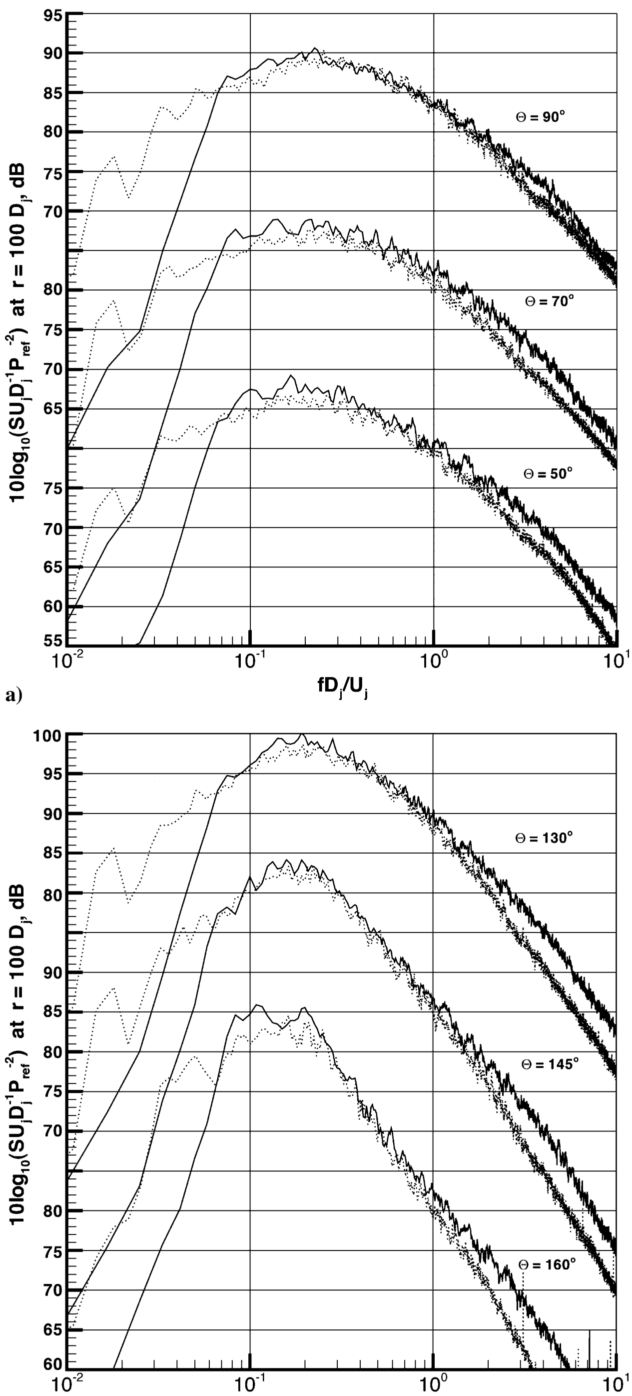


Fig. 3 Scaling of jet-noise spectra. Boeing data: $M_j = 0.5$; $T_r/T_\infty = 2.15$; ——, $D_j = 3.46$ in. (8.79 cm); and · · · ·, $D_j = 1.5$ in. (3.81 cm).

the Reynolds numbers of these jets. There is no question that, as shown in Table 1, the Reynolds number is low at temperature ratio 3.2 for the smallest jets. However, in Table 2 at temperature ratio 1.8 and Mach 1.0, the Reynolds number of the smallest jet is more than half a million. As yet, Fig. 6 shows that the noise spectra still have discrepancies at high Strouhal number similar to those for the lower Reynolds number jet. Here we will not pursue this further as it may be a facility problem or even a humidity correction problem.

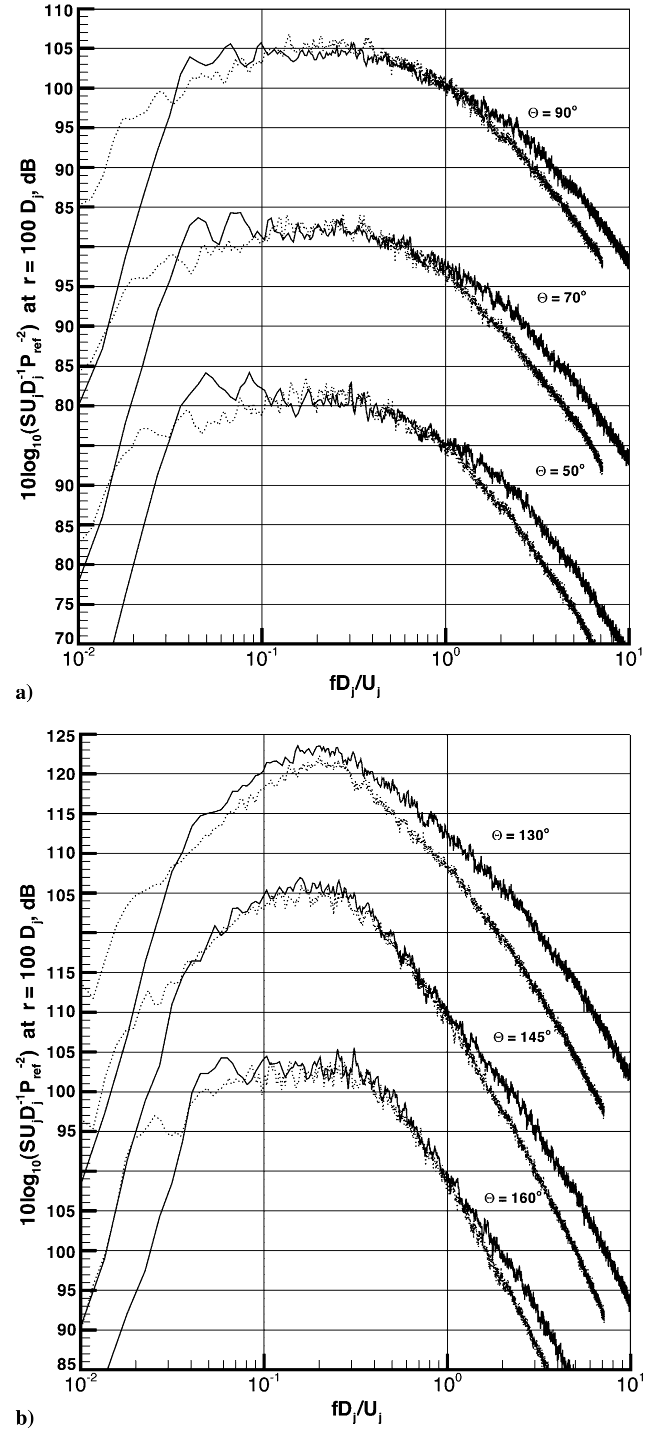


Fig. 4 Scaling of jet-noise spectra. Boeing data: $M_j = 1.0$; $T_r/T_\infty = 2.15$; ——, $D_j = 3.46$ in. (8.79 cm); and · · · ·, $D_j = 1.5$ in. (3.81 cm).

Humidity correction starts to become important at 20 KHz. For the smallest jet, the corresponding Strouhal number lies in the range of 1.3–2.7. If there is an overcorrection, the noise spectra would be lower at Strouhal number higher than the 20 KHz Strouhal number. We have no evidence, however, that this is the case.

So far, we have demonstrated good data scaling using data from the same investigator. Let us now try to scale data together from multiple sources. Figure 9 shows the spectra of noise data from Refs. 9 and 23 and the Boeing data plotted according to scaling formula (4). As can be seen, there is general agreement among the three sets of data except at very low and very high frequencies. Our speculation is that most of the differences appear to be a facility and instrumentation problem. It would definitely be helpful, especially to investigators who intend to develop jet-noise prediction theories,

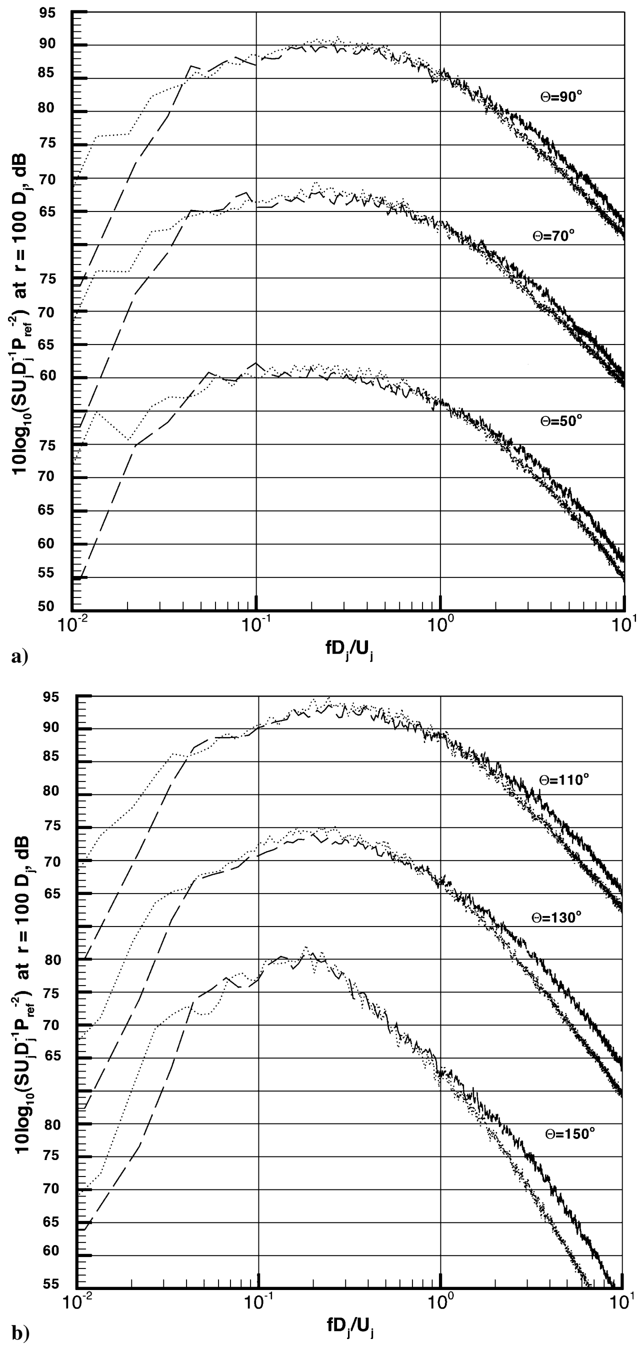


Fig. 5 Scaling of jet-noise spectra. Data from Vishwanathan[9]: $M_j = 0.6$; $T_r/T_\infty = 1.8$; ‒‒‒, $D_j = 2.45$ in. (6.22 cm); and · · · ·, $D_j = 1.5$ in. (3.81 cm).

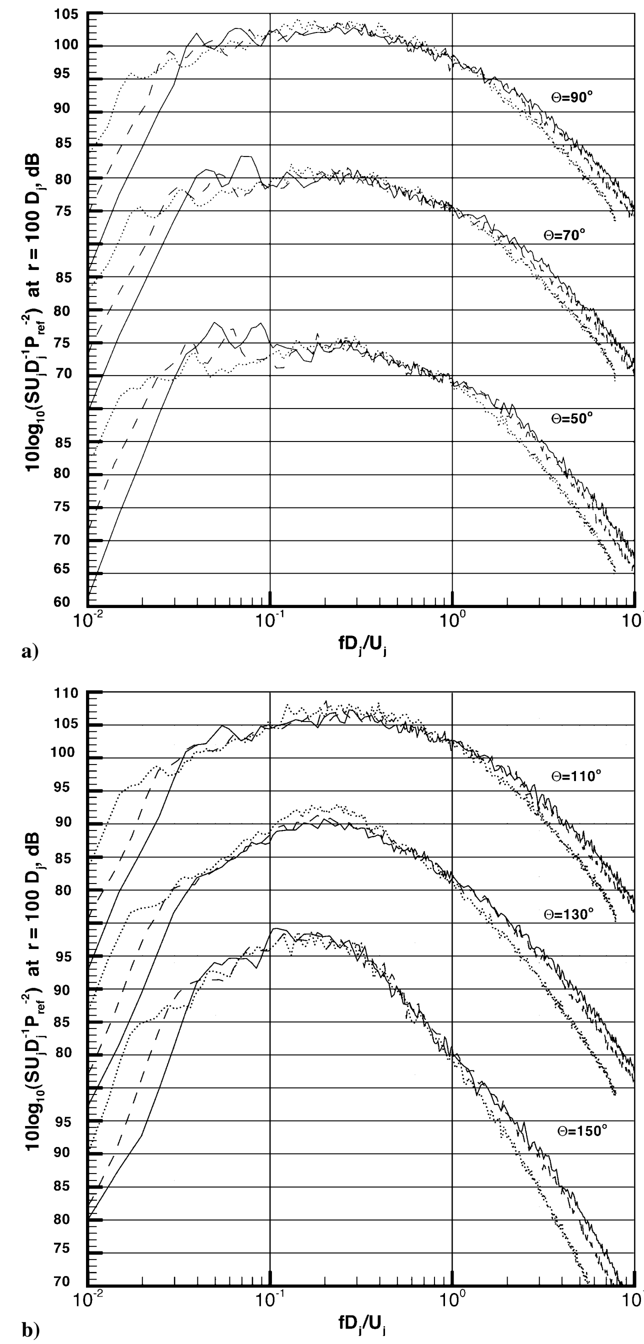


Fig. 6 Scaling of jet-noise spectra. Data from Vishwanathan[9]: $M_j = 1.0$; $T_r/T_\infty = 1.8$; ——, $D_j = 3.46$ in. (8.79 cm); ‒‒‒, $D_j = 2.45$ in. (6.22 cm); and · · · ·, $D_j = 1.5$ in. (3.81 cm).

that data from different facilities can be accurately scaled together. Such collapsed spectra, when available, would be benchmark data for testing theoretical predictions.

IV. Power Law

It has been known since the early days of jet-noise research that the noise intensity varies as a high power of the jet velocity. This power law has the support of both theory and experiment. The most celebrated power law is the Lighthill U^8 law. Numerous experiments have been performed for the purposes of verifying the U^8 law. Some measurements indicate that the exponent is close to but not precisely equal to 8. A more general power law is sometimes written in the form

$$I \propto U_j^n \tag{5}$$

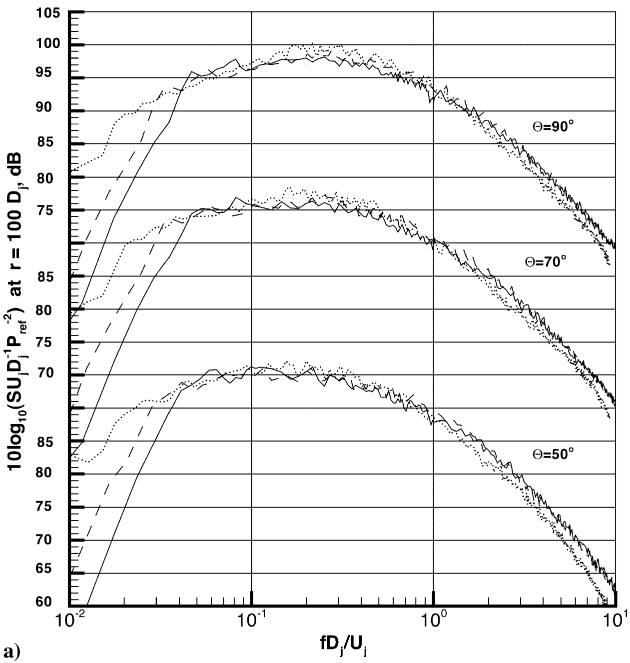
In a recent experiment, Viswanathan[9] confirmed that jet-noise intensity could, indeed, be correlated with a power law of the form (5). He found that n depends on jet temperature. It varied from 7.98 to 8.74.

A power law in the form of (5) is dimensionally unsatisfactory. In this section, our goal is to express the power law in a dimensionless form and then examine the dependence of its parameters on the dimensionless groups of the input variables, as discussed in Sec. II.

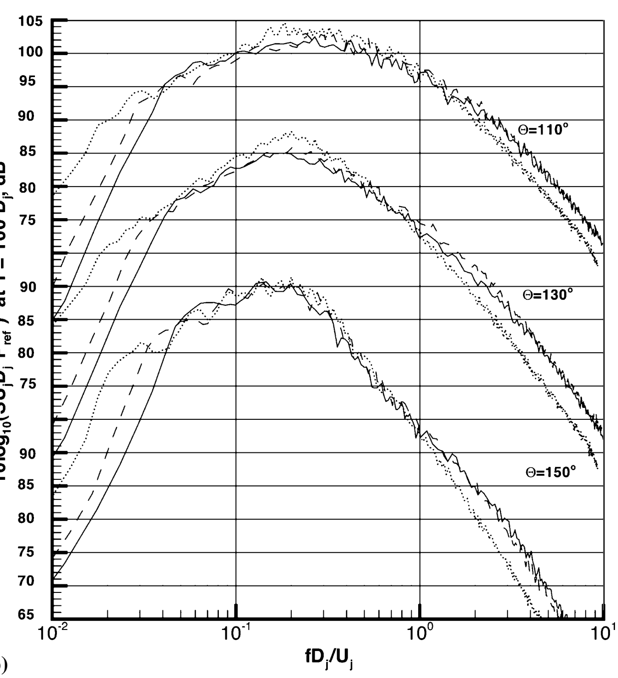
Let us define the noise intensity (or overall sound pressure level) I for noise radiation in the far-field direction θ as

$$I = \int_0^\infty S(r, \theta, f)\,\mathrm{d}f \tag{6}$$

Note that the noise intensity defined by Eq. (6) differs from the traditional definition by a factor of $\rho_\infty a_\infty$. In Eq. (6) S is the

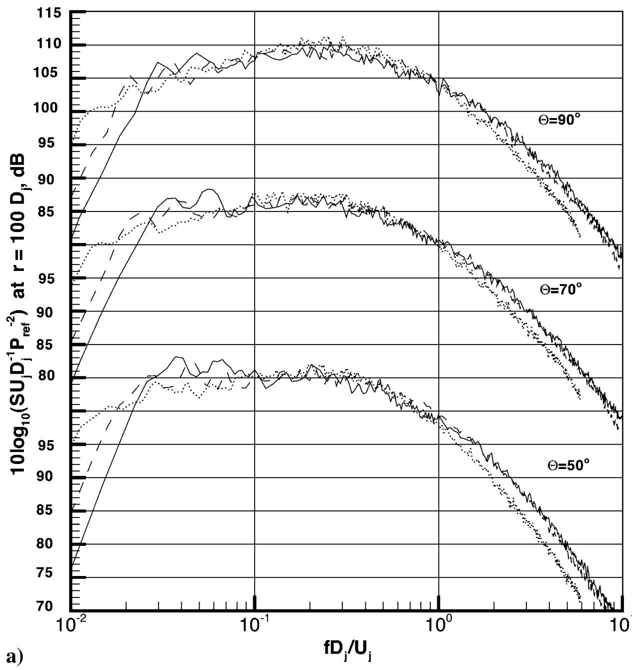


a)

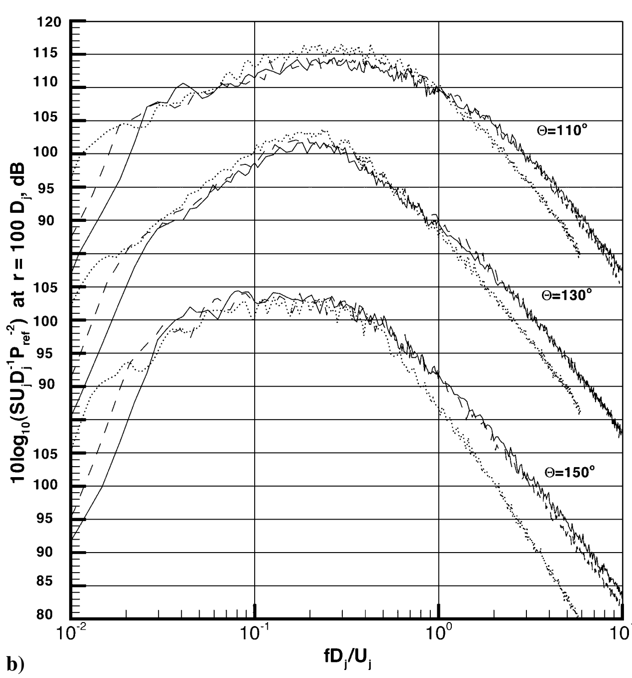


b)

Fig. 7 Scaling of jet-noise spectra. Data from Vishwanathan[9]: $M_j = 0.6$; $T_r/T_\infty = 3.2$; ——, $D_j = 3.46$ in. (8.79 cm); – – –, $D_j = 2.45$ in. (6.22 cm); and · · · ·, $D_j = 1.5$ in. (3.81 cm).



a)



b)

Fig. 8 Scaling of jet-noise spectra. Data from Vishwanathan[9]: $M_j = 1.0$; $T_r/T_\infty = 3.2$; ——, $D_j = 3.46$ in. (8.79 cm); – – –, $D_j = 2.45$ in. (6.22 cm); and · · · ·, $D_j = 1.5$ in. (2.81 cm).

spectral density. Now Eq. (6) may be rewritten in a dimensionless form as

$$\frac{I}{p_\infty^2} = \int_0^\infty \frac{S(r/D_j, \theta, fD_j/U_j, U_j/a_\infty, T_r/T_\infty)U_j}{p_\infty^2 D_j} \mathrm{d}\left(\frac{fD_j}{U_j}\right) \tag{7}$$

On replacement of the integrand by Eq. (3), Eq. (7) becomes

$$\frac{I}{p_\infty^2} = \int_0^\infty \frac{F(U_j/a_\infty, T_r/T_\infty, fD_j/U_j, \theta)}{(r/D_j)^2} \mathrm{d}\left(\frac{fD_j}{U_j}\right) = \frac{K(U_j/a_\infty, T_r/T_\infty, \theta)}{(r/D_j)^2} \tag{8}$$

Experimental observations suggest that the function K may be expressed as a high power of (U_j/a_∞). That is,

$$\frac{I(r,\theta)}{p_\infty^2} \cong \frac{A(U_j/a_\infty)^n}{(r/D_j)^2} \tag{9}$$

where A and n are two free parameters of the power law. They depend on the temperature ratio and the direction of radiation; i.e., $A = A(T_r/T_\infty, \theta)$, $n = n(T_r/T_\infty, \theta)$.

The validity of the power law in the form of Eq. (9) can be confirmed experimentally. Figure 10 shows a log–log plot of I/p_∞^2 at $r/D = 100.0$ as a function of U_j/a_∞ for jet noise radiated in the 90-deg direction based on the data of Viswanathan[9] and Seiner (see Ref. 22). Each of the data sets is for a fixed temperature ratio. The arrow marks the value −80.0 in the vertical axis of Fig. 10 for the particular curve. It is clear that, to a good degree of approximation, a

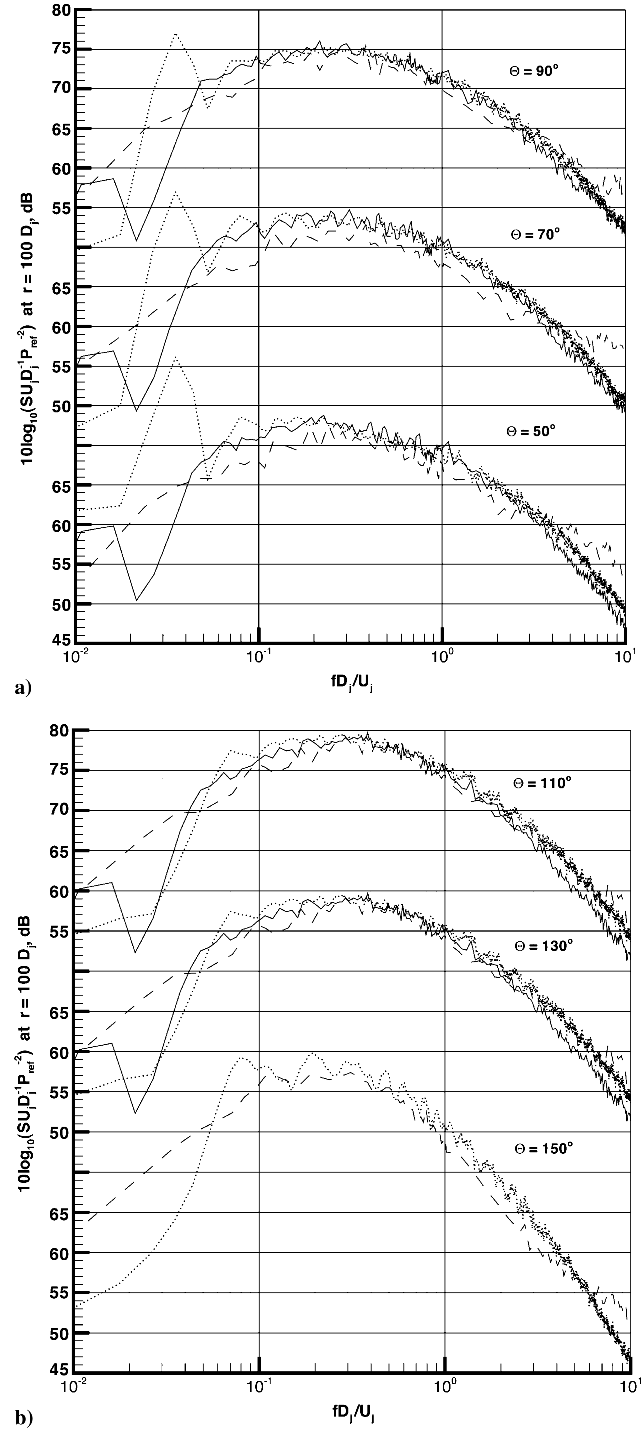


Fig. 9 Scaling of jet-noise spectra: $M_j = 0.5$; $T_r/T_\infty = 1.0$; ——, Boeing data, $D_j = 1.5$ in. (3.81 cm); – – –, Norum and Brown,[23] $D_j = 0.75$ in. (1.91 cm); and · · · ·, Vishwanathan,[9] $D_j = 2.45$ in. (6.22 cm).

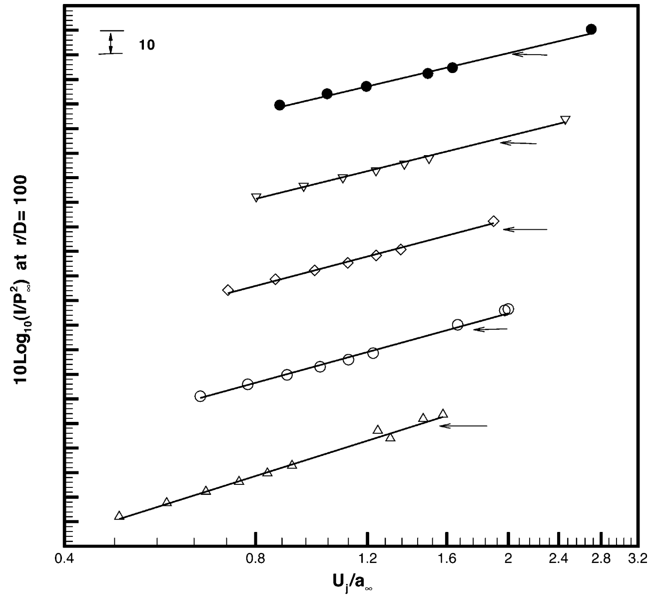


Fig. 10 Dependence of noise intensity on jet Mach number based on ambient speed of sound. Inlet angle 90 deg. ← vertical axis equals −80.0 for the curve: ●, ▽, ◇, ○, △, $T_r/T_\infty = 1.0$, 1.8, 2.2, 2.7, 3.2; ——, power law A $(U_j/a_\infty)^n/(r/D)^2$. Data from Vishwanathan[9] and Tam et al.[22]

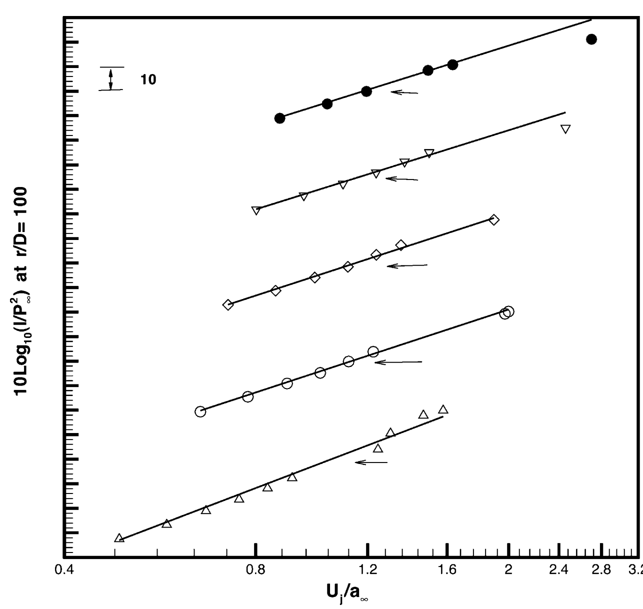


Fig. 11 Dependence of noise intensity on jet Mach number based on ambient speed of sound. Inlet angle 150 deg. ← vertical axis equals −80.0 for the curve: ●, ▽, ◇, ○, △, $T_r/T_\infty = 1.0$, 1.8, 2.2, 2.7, 3.2; ——, power law A $(U_j/a_\infty)^n/(r/D)^2$. Data from Vishwanathan[9] and Tam et al.[22]

linear relationship exists in the log–log plot. This validates a power-law relationship as given in Eq. (9). Figure 11 shows a similar plot for noise radiated in the 150-deg inlet-angle direction. Again a linear relationship between I/p_∞^2 and U_j/a_∞ exists at each temperature ratio. In addition to the data shown in Figs. 10 and 11, we have examined data at other angles (from 50 to 150 deg). All of them can be correlated with a power law. We believe this provides strong empirical support for the general validity of Eq. (9).

The exponent n of power law (9), which is the slope of the straight lines in Figs. 10 and 11, varies slightly with jet-temperature ratio and the direction of radiation. The same is true with the coefficient A. Figure 12 gives the dependence of n on the temperature ratio (T_r/T_∞) for a number of directions of radiation. Generally speaking, n is large for larger inlet angle. Its value decreases with increased temperature ratio. Numerically, n varies from 5.3 to 9.9; $n = 8$ may be considered as an overall averaged value.

The dependence of coefficient A in power law (9) on jet-temperature ratio is given in Fig. 13. Just as for exponent n, the value of A is larger for larger inlet angle. Also, its value decreases nearly monotonically with increased temperature ratio.

It is interesting to point out that as shown in Figs. 12 and 13, n and A decrease monotonically as temperature ratio T_r/T_∞ increases. However, both n and A cannot become zero. Thus, it is reasonable to expect that as temperature ratio becomes sufficiently large, n and A will each independently reach an asymptotic value. Figures 12 and 13 suggest that at the last data point $(T_r/T_\infty = 3.2)$ both curves for

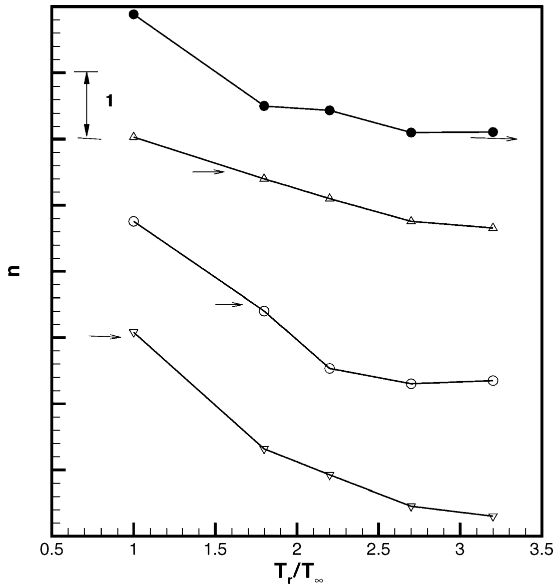


Fig. 12 Variation of power-law exponent with jet-temperature ratio. ← vertical axis equals 8.0 for the curve. ▽, ○, △, ●: inlet angle = 50, 80, 120, and 150 deg. Data from Vishwanathan[9] and Tam et al.[22]

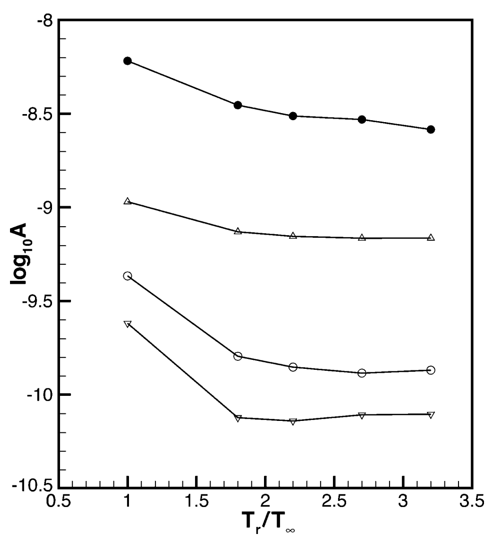


Fig. 13 Variation of power-law proportionality constant with jet-temperature ratio. ▽, ○, △, ●; inlet angle = 50, 80, 120, and 150 deg. Data from Vishwanathan[9] and Tam et al.[22]

n and A are close to their asymptotes. We will call this the "hot-jet" limit. The physical significance of the hot-jet limit is that for jets at high temperature ratio, the noise intensity depends only on the jet velocity ratio. Temperature plays no role in jet-noise intensity except through its influence on the nozzle exit velocity. This is an unexpected result. Of course, it would require further investigation before one could determine what the hot-jet limit meant to the distribution of jet-noise sources in the jet plume and in the jet mixing processes.

V. Seasonal and Altitude Variation of Jet Noise

For some time, it has been known to investigators at the GE Aircraft Engines Co. that there is a seasonal variation of jet noise.[24] For the same jet exit velocity, there is more jet noise in winter than in summer. Experimental data showing the seasonal change of jet noise are shown in Fig. 14. Roughly speaking, there is about a 2-EPNdB decrease in noise level as the season changes from winter to summer. Here we seek an explanation of this phenomenon.

Power law (9) gives the dependence of jet-noise intensity on environmental parameters as well as jet exit velocity and temperature.

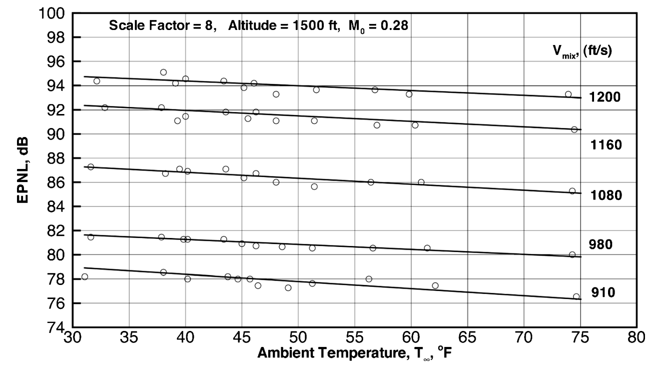


Fig. 14 EPNL as a function of T_∞ for baseline BPR = 5 nozzle (3BB) (Ref. 24) at 1500 ft = 457.2 m; V_{mix} is the average jet velocity (ft/s × 0.305 = m/s).

Let us assume that the ambient static pressure, p_∞, remains constant and concentrate on studying the effect of variation of ambient temperature T_∞ alone. Consider an experiment performed in winter at $T_\infty = 30°\text{F}$ (−1.1°C). The same experiment is repeated in summer at the same jet exit velocity at $T_\infty = 75°\text{F}$ (23.9°C). Over this range of temperature, the exponent n and coefficient A of power law (9) remains practically the same. For simplicity, we will assume they remain unchanged. By means of (9), it is easy to derive

$$\frac{I_{75°\text{F}}}{I_{30°\text{F}}} = \left[\frac{(a_\infty)_{30°\text{F}}}{(a_\infty)_{75°\text{F}}}\right]^n = \left[\frac{T_{30°\text{F}}}{T_{75°\text{F}}}\right]^{n/2} = 0.957^n$$

Therefore

$$\Delta\text{dB} = 10\log(I_{75°\text{F}}) - 10\log(I_{30°\text{F}}) = 10n\log(0.957)$$

On taking n to be 8, we find that $\Delta\text{dB} = -1.53$. Here we expect a decrease in noise intensity of around 1.5 dB in summer. This is consistent with the experimental data of Fig. 14. Formula (9) does not provide spectral information, so it is not possible to estimate the noise-level decrease in EPNdB units.

The change in ambient temperature would, invariably alter to some degree, the mixing between the jet and ambient fluid. In this way, it would affect the noise sources of the jet. At the same time, changing the ambient temperature and hence the gas density would affect the impedance and the propagation of sound from the source in the jet to a far-field observer. Now we wish to see if it is possible to determine the contribution of each of these two effects to the phenomenon of summer noise reduction. A detailed investigation is beyond the scope of this work. However, for fine-scale turbulent jet noise, there is a fully developed theory by Tam and Auriault.[25] Let us begin with this theory and see what can be learned. For simplicity, we will consider noise radiation to 90-deg inlet angle. At 90-deg radiation, there are no source convection or mean flow refraction effects. The formula for the radiated noise spectrum of a jet given by Tam and Auriault is

$$S(\boldsymbol{x}, \omega) = 4\pi\left(\frac{\pi}{\ln 2}\right)^{\frac{3}{2}} \iiint\limits_{\text{volume of jet}} \frac{\hat{q}_s^2 \ell_s^3}{c^2 \tau_s} \times \frac{|p_a(\boldsymbol{x}_2, \boldsymbol{x}, \omega)|^2 \, e^{-\omega^2 \ell_s^2/\bar{u}^2(4\ln 2)}}{\left(1 + \omega^2 \tau_s^2\right)} \, \mathrm{d}\boldsymbol{x}_2 \tag{10}$$

where $\omega = 2\pi f$ and $p_a(\boldsymbol{x}_2, \boldsymbol{x}, \omega)$ is the adjoint Green's function.

The adjoint Green's function contains all propagation and impedance effects. For 90-deg radiation, the adjoint Green's function outside the jet has the form

$$p_a(\boldsymbol{x}_2, \boldsymbol{x}, \omega) = -\frac{i\omega}{8\pi^2 a_\infty^2} \frac{e^{i(\omega/a_\infty)|\boldsymbol{x}-\boldsymbol{x}_2|}}{|\boldsymbol{x} - \boldsymbol{x}_2|} \tag{11}$$

Table 3 ΔdB for various airports[a]

City	Height, ft (m)	p_∞, psi	T_∞, F°(C°)	ΔdB Pressure	ΔdB Temperature	ΔdB
Denver	5431 (1665)	12.00	38.7 (3.7)	−1.76	+0.69	−1.07
Salt Lake	4226 (1228)	12.56	43.9 (6.6)	−1.37	+0.51	−0.86
Atlanta	1026 (313)	14.17	55.4 (13)	−0.32	+0.12	−0.20

[a]Standard model: $p_{\text{sea level}} = 14.7$ psi, $T_{\text{sea level}} = 59°$F or 15°C; $n = 8$.

Therefore,

$$|p_a|^2 = \left(\omega^2 / 64\pi^4 a_\infty^4\right)\left(1 / |\boldsymbol{x} - \boldsymbol{x}_2|^2\right) \tag{12}$$

In other words, impedance and propagation effects account for approximately an inverse fourth power of a_∞. This is about half the dependence of I on a_∞^{-1}. So we may conclude that approximately half of the ambient temperature effect on radiated jet noise comes from changes in noise source strength and the other half from impedance and propagation effects.

Other than seasonal variation, power law (9) allows us to estimate noise-level variation arising from the location (altitude) of an airport. There is a change in ambient pressure and temperature with altitude. Based on this observation, we might expect a slight decrease in community noise around Denver International Airport, which is about a mile high, as compared to, say, Los Angeles International Airport, which is at sea level. Starting from power law (9), it is straightforward to derive the following altitude variation formula for jet noise intensity:

$$\Delta\text{dB} = 10\log\left(\frac{I_{\text{airport}}}{I_{\text{sea level}}}\right) = 10\log\left(\frac{p^2_{\infty\,\text{airport}}}{p^2_{\infty\,\text{sea level}}}\right) + 5n\log\left(\frac{T_{\text{sea level}}}{T_{\text{airport}}}\right) \tag{13}$$

Let us use (13) and the standard atmospheric pressure and temperature model for altitude variation to estimate the ΔdB values. Table 3 gives the ΔdB values for Denver, Salt Lake, and Atlanta airports.

The ΔdB values in Table 3 comprise two parts. One component is because of the change in ambient pressure and the other component is due to change in ambient temperature. The two effects tend to cancel each other, making the total ΔdB smaller. The absolute value is small. The largest ΔdB is for Denver International Airport. It has a noise-intensity reduction of about 1.1 dB.

VI. Imperfectly Expanded Supersonic Jets

For imperfectly expanded supersonic jets, the pressure at the nozzle exit is not the same as the ambient pressure. Thus the analysis of Sec. II is not directly applicable to these jets. Let us consider an underexpanded jet from a convergent–divergent (C–D) nozzle, as shown in Fig. 15. By definition the static pressure of the jet at the nozzle exit is higher than the ambient pressure. The gas coming out of the nozzle would, therefore, undergo further expansion. This further expansion leads to an effective (or fully expanded) jet diameter D_j larger than the physical diameter D of the nozzle exit. A simple relationship between D_j and D can be found by appealing to conservation of mass flux.

Since the gas expands downstream of the nozzle exit, it is therefore a reasonably good approximation to assume that the flow through the nozzle as well as in the initial portion of the free jet is isentropic. We will use subscripts r, e, and j to denote variables in the reservoir, at the nozzle exit and in the fully developed jet. Conservation of enthalpy leads to

$$C_p T_r = C_p T_e + \tfrac{1}{2}U_e^2 = C_p T_j + \tfrac{1}{2}U_j^2 \tag{14}$$

where C_p is the specific heat at constant pressure. Equation (14) may be rewritten as

$$T_r/T_e = 1 + [(\gamma - 1)/2]M_d^2 = (\rho_r/\rho_e)^{\gamma-1} \tag{15}$$

$$T_r/T_j = 1 + [(\gamma - 1)/2]M_j^2 = (\rho_r/\rho_j)^{\gamma-1} \tag{16}$$

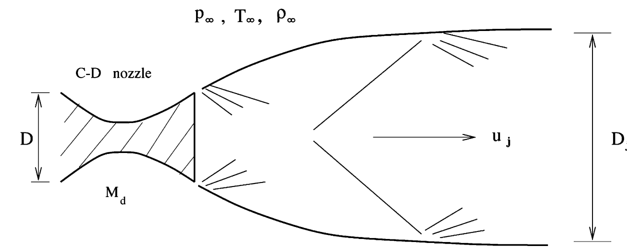


Fig. 15 Schematic diagram of an underexpanded supersonic jet.

The last equality of Eqs. (15) and (16) comes from the isentropic relation $T \propto \rho^{\gamma-1}$. In (15) M_d is the design Mach number of the C–D nozzle ($M_d = 1.0$ for a convergent nozzle) and is equal to the flow Mach number at the nozzle exit; i.e., $M_d = M_e$. M_j in (16) is the fully expanded jet Mach number. The combination of (15) and (16) yields

$$\frac{\rho_e}{\rho_j} = \left[\frac{1 + [(\gamma - 1)/2]M_j^2}{1 + [(\gamma - 1)/2]M_d^2}\right]^{1/(\gamma-1)} \tag{17}$$

$$\frac{T_j}{T_e} = \left[\frac{1 + [(\gamma - 1)/2]M_d^2}{1 + [(\gamma - 1)/2]M_j^2}\right] \tag{18}$$

By conservation of mass flux, we have

$$\rho_e U_e D^2 = \rho_j U_j D_j^2 \tag{19}$$

Thus, by using Eqs. (14) and (17–19), we find

$$\begin{aligned}\frac{D_j}{D} &= \left(\frac{\rho_e}{\rho_j}\right)^{\frac{1}{2}}\left(\frac{U_e}{U_j}\right)^{\frac{1}{2}} \\ &= \left[\frac{1 + [(\gamma - 1)/2]M_j^2}{1 + [(\gamma - 1)/2]M_d^2}\right]^{1/2(\gamma-1)}\left(\frac{M_d}{M_j}\right)^{\frac{1}{2}}\left(\frac{T_e}{T_j}\right)^{\frac{1}{4}} \\ &= \left(\frac{M_d}{M_j}\right)^{\frac{1}{2}}\left[\frac{1 + [(\gamma - 1)/2]M_j^2}{1 + [(\gamma - 1)/2]M_d^2}\right]^{(\gamma+1)/4(\gamma-1)}\end{aligned} \tag{20}$$

Formula (20) was first stated in the work of Tam and Tanna[26] but without a detailed derivation.

We want to point out that for an overexpanded jet, the jet fluid will encounter shocks immediately downstream of the nozzle exit. However, for moderately overexpanded jets, the shocks are relatively weak. The change in entropy across the weak shocks of these jets is of higher order in the jump in Mach number. The flow, therefore, does not deviate badly from isentropic flow. For this reason, Eq. (20) may still be useful for estimating the fully developed jet diameter of moderately overexpanded jets.

Shock-containing jets usually emit screech tones[27] driven by a feedback phenomenon. For strongly screeching jets, the flow dynamics are substantially changed by the feedback. Here we consider jets that have only mild screech tones.

Now as far as jet-mixing noise is concerned, D_j, given by Eq. (20), is the pertinent length scale. Thus the results obtained by dimensional analysis in Sec. II would apply to even imperfectly expanded supersonic jets if D_j instead of D were used in the scaling formula (3). To demonstrate the applicability of scaling formula (3) with this modification, we will show that the turbulent mixing noise of an underexpanded jet can be scaled to that of a perfectly expanded jet by means of formula (3). Figure 16 shows two sets of jet-noise spectra measured by Seiner and coworkers (see Ref. 22) at fully expanded Mach number 1.5. One set of spectra is from a jet issued from a convergent nozzle ($M_d = 1.0$). The other set is from a nearly perfectly expanded jet ($M_d = 1.5$). As can be seen, by using the fully expanded jet diameter D_j provided by Eq. (20), there is a good collapse of the data. The underexpanded jet has additional broadband

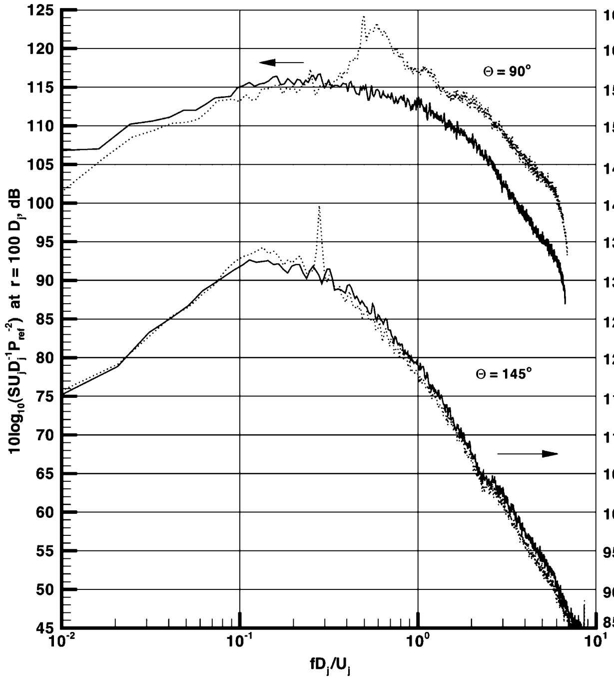


Fig. 16 Scaling of perfectly and imperfectly expanded supersonic jet-noise spectra, $M_j = 1.5$: ——, $M_d = 1.5$, $D = 1.68$ in. (4.27 cm); · · · ·, $M_d = 1.0$, $D = 1.56$ in. (3.96 cm). $T_r/T_\infty = 2.23$ for inlet angle 90 deg; $T_r/T_\infty = 1.45$ for inlet angle 145 deg.

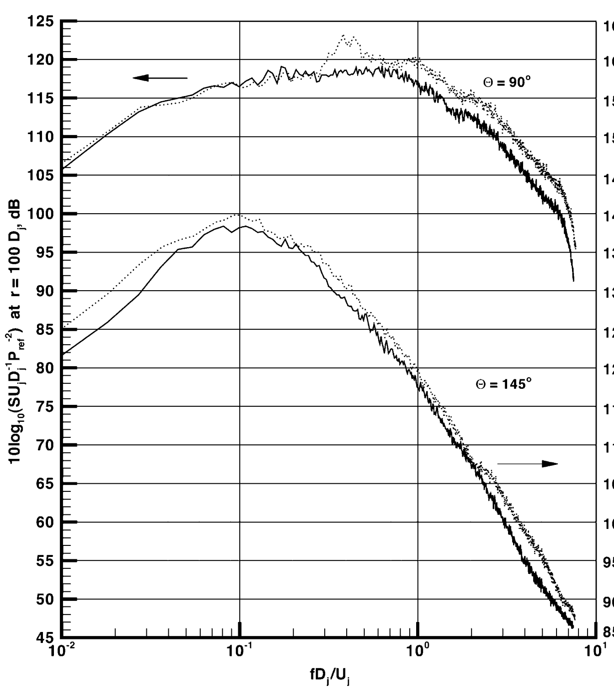


Fig. 17 Scaling of perfectly and imperfectly expanded supersonic jet-noise spectra for $M_j = 2.0$, $T_r/T_\infty = 1.8$: ——, $M_d = 2.0$, $D = 1.96$ in. (4.99 cm); · · · ·, $M_d = 1.0$, $D = 1.56$ in. (3.96 cm).

shock–associated noise at high Strouhal number at $\Theta = 90$ deg but not at $\Theta = 145$ deg. Figure 17 shows similar data scaling for Mach 2 jets. In this case, D_j is much larger than D for underexpanded jets. The spectra collapse well, using D_j of Eq. (20) as the length scale. The data level will differ by several dB if D is used instead.

VII. Summary

In this paper, the development of a jet-noise scaling formula is reported. The scaling formula is developed through dimensional analysis and the Buckingham Π theorem. To demonstrate the usefulness of this formula, it is applied to a large set of data measured over a large range of jet Mach number and temperature ratio. Good data collapse, within experimental accuracy, is found.

It is well known that jet-noise intensity varies as a high power of jet exit velocity (fully expanded jet velocity for supersonic jets). The most famous power law is the Lighthill U^8 law. In this work, a properly nondimensional form of the generalized power law is established. An examination of the temperature-ratio dependence of the parameters of the generalized power law reveals the existence of a hot jet limit. In this limit, the radiated noise is insensitive to the jet-temperature ratio and depends only on the jet Mach number, based on ambient sound speed. The generalized power law allows an assessment of seasonal variation of jet noise. A simple estimate suggests a summer reduction of noise intensity compared with winter measurements of about 1.5 dB. This is in good agreement with experimental measurements. Altitude variation of jet noise is investigated by means of the generalized power law. It is found that airports located at a high altitude would experience a noise intensity reduction when compared to airports located at sea level.

For imperfectly expanded supersonic jets, the nozzle exit diameter is not the proper length scale of the mixing noise problem. A formula for calculating the fully expanded jet diameter is provided. It is shown that by using this fully expanded jet diameter as the length scale, the mixing noise spectrum of an imperfectly expanded jet can be scaled to that of a perfectly expanded jet.

Recently quality issues related to jet-noise data have received increasing attention.[18] As yet, how to evaluate the accuracy of a set of jet-noise data remains an outstanding question. One way to ensure accuracy, perhaps, is to establish a set of widely accepted standard jet-noise spectra. Such a set of standard spectra can be obtained by comparing data from different experimental facilities and by collapsing noise data from jets of different sizes. Such an effort is badly needed. Here an appeal is made to the jet aeroacoustics community. To drive home that such a community effort is necessary, we would like to refer to our effort of trying to collapse data from multiple sources, as shown in Fig. 9. There is a scattering of data as large as 5 dB at specific Strouhal numbers. This discrepancy is too large for design purpose. It is hoped that our appeal will help to stimulate action in the development of standard jet-noise spectra. This is best done as a community effort.

Jet-noise research has gone on for over a half century, but we still do not know which set of jet-noise data is accurate. This state of affairs, from the present author's viewpoint, requires immediate corrective actions.

Acknowledgments

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